International Journal of Scientific and Innovative Mathematical Research (IJSIMR)
Volume 3, Special Issue 2, July 2015, PP 434-439, ISSN 2347-307X (Print)
www.arcjournals.org

LEFT GENERALIZED JORDAN DERIVATIONS ON PRIME RINGS

C. Jaya Subba Reddy[1], S.MallikarjunaRao[2], K.Madhusudhan Reddy[3]
[1,2]Department of Mathematics, S.V.University, Tirupati, AndhraPradesh, India.
[3]School of Advanced Sciences, VIT University ,Vellore,India.

Abstract*: In this paper, we proved that left generalized Jordan derivation and left generalized Jordan triple derivation on 2-torsion free prime ring is a left generalized derivation.*

Keywords:*Prime ring, Derivation, Generalized derivation, Left generalized derivation,Left generalized Jordan derivation and Left generalized Jordan triple derivation.*

INTRODUCTION AND PRELIMINARIES

Throughout this paper R will be represent an associative ring with center $Z(R)$. Recall that R is a prime if $aRb = 0$ implies that either $a = 0$ or $b = 0$ and is semiprime if $aRa = 0$ implies $a = 0$. An additive map $d: R \to R$ is called a derivation of R if $d(xy) = d(x)y + x\,d(y)$ for all x, y in R. An additive map $F: R \to R$ is called a generalized derivation if there exists a derivation $d: R \to R$ such that $F(xy) = F(x)y + x\,d(y)$ for all $x, y \in R$. An additive map $F: R \to R$ is called a left generalized derivation if there exists a derivation $d: R \to R$ such that $F(xy) = d(x)y + x\,F(y)$ for all $x, y \in R$. An additive map $\delta: R \to R$ is called a left generalized Jordan derivation if there exists a derivation $\tau: R \to R$ such that $\delta(x^2) = \tau(x)x + x\,\delta(x)$ for all $x, y \in R$. An additive map $\delta: R \to R$ is called a left generalized Jordan triple derivation if there exists a derivation $\tau: R \to R$ such that $\delta(abc) = \tau(a)bc + a\,\tau(b)c + ab\delta(c)$ for all $a, b, c \in R$. The concept of generalized derivations, Jordan derivations and Jordan triple derivation was introduced by Hvala [4] and Bresar [1]. Generalized Jordan derivation defined by J.Zhu and C.Xiong[5]. Wu jing and shijie lu [6] studied generalized Jordan derivations on prime rings and standard operator algebras . In this paper we proved some results of left generalized Jordan derivation and left generalized Jordan triple derivation in 2-torsion free prime ring is a left generalized derivation.

Main results:

Lemma1: $For\ all\ a, b, c \in R\ the\ following\ statements\ hold$:

$$i)\delta(ab + ba) = \tau(a)b + a\delta(b) + \tau(b)a + b\delta(a);$$
$$ii)\delta(aba) = \tau(a)ba + a\tau(b)a + ab\delta(a);$$
$iii)\delta(abc + cba) = \tau(a)bc + a\tau(b)c + ab\delta(c) + \tau(c)ba + c\tau(b)a + cb\delta(a)$.

Proof: $i)\delta(a + b)^2 = \delta((a + b)(a + b)) = \tau(a + b)(a + b) + (a + b)\delta(a + b)$
$= \tau(a)a + \tau(a)b + \tau(b)a + \tau(b)b + a\delta(a) + a\delta(b) + b\delta(a) + b\delta(b)$.
$\delta(a + b)^2 = \tau(a)a + \tau(a)b + \tau(b)a + \tau(b)b + a\delta(a) + a\delta(b) + b\delta(a) + b\delta(b)$. (1)
On the other hand, we have
$\delta(a + b)^2 = \delta((a + b)(a + b))$
$=\delta(a^2 + ab + ba + b^2) = \delta(a^2) + \delta(ab + ba) + \delta(b^2)$
$= \tau(a)a + a\delta(a) + \delta(ab + ba) + \tau(b)b + b\delta(b)$.
$\delta(a + b)^2 = \tau(a)a + a\delta(a) + \delta(ab + ba) + \tau(b)b + b\delta(b)$. (2)
From (1) &(2),we have
$\tau(a)a + \tau(a)b + \tau(b)a + \tau(b)b + a\delta(a) + a\delta(b) + b\delta(a) + b\delta(b)$
$= \tau(a)a + a\delta(a) + \delta(ab + ba) + \tau(b)b + b\delta(b)$
$\delta(ab + ba) = \tau(a)b + a\delta(b) + \tau(b)a + b\delta(a)$.
ii) Let $W = \delta(a(ab + ba) + (ab + ba)a)$.
On one hand, we have
$W = \tau(a)(ab + ba) + a\delta(ab + ba) + \tau(ab + ba)a + (ab + ba)\delta(a)$

$W = \tau(a)ab + \tau(a)ba + a\big(\tau(a)b + a\delta(b) + \tau(b)a + b\delta(a)\big) + \big(\tau(a)b + a\tau(b) + \tau(b)a + b\tau(a)\big)a + ab\delta(a) + ba\delta(a)$

$W = \tau(a)ab + \tau(a)ba + a\tau(a)b + a^2\delta(b) + a\tau(b)a + ab\delta(a)) + (\tau(a)b + a\tau(b) + \tau(b)a + b\tau(a)\,)a + ab\delta(a) + ba\delta(a).$ (3)

On the other hand, we have

$W = \delta(a^2b + 2aba + ba^2).$

$W = \tau(a^2)b + a^2\delta(b) + 2\delta(aba) + \tau(b)a^2 + b\delta(a^2)$

$W = (\tau(a\,)a + a\tau(a))b + a^2\delta(b) + 2\delta(aba) + \tau(b)a^2 + b(\tau(a)a + a\delta(a))$

$W = \tau(a\,)ab + a\tau(a)b + a^2\delta(b) + 2\delta(aba) + \tau(b)a^2 + b\tau(a)a + ba\delta(a).$ (4)

From (3) & (4), we have

$\tau(a)ab + \tau(a)ba + a\tau(a)b + a^2\delta(b) + a\tau(b)a + ab\delta(a)) + \tau(a)ba + a\tau(b)a + \tau(b)a^2 + b\tau(a)a + ab\delta(a) + ba\delta(a)$
$= \tau(a\,)ab + a\tau(a)b + a^2\delta(b) + 2\delta(aba) + \tau(b)a^2 + b\tau(a)a + ba\delta(a)$

$2\delta(aba) = 2\tau(a)ba + 2a\tau(a)b + 2\,ab\delta(a)$

Since R is 2-torsion free, we get

$\delta(aba) = \tau(a)ba + a\tau(a)b + ab\delta(a).$

(iii) Linearizing (ii) by replacing a by $a + c$

$\delta\big((a+c)b(a+c)\big) = \tau(a+c)b(a+c) + (a+c)\tau(b)(a+c) + (\,a+c)b\delta(a+c)$

From L.H.S

$\delta\big((a+c)b(a+c)\big) = \delta\big((ab+cb)(a+c)\big)$

$= \delta(aba + abc + cba + cbc)$

$= \delta(aba) + \delta(abc) + \delta(cba) + \delta(cbc)$

$= \tau(a)ba + a\tau(b)a + ab\delta(a) + \delta(abc + cba) + \tau(c)bc + c\tau(b)c + cb\delta(c)$

$\delta\big((a+c)b(a+c)\big) = \tau(a)ba + a\tau(b)a + ab\delta(a) + \delta(abc + cba) + \tau(c)bc + c\tau(b)c + cb\delta(c)\,.$ (5)

From R.H.S

$\tau(a+c)b(a+c) + (a+c)\tau(b)(a+c) + (\,a+c)b\delta(a+c)$

$= (\tau(a) + \tau(c))(ba + bc) + (a\tau(b) + c\tau(b))(a+c) + (\,ab + cb)(\delta(a) + \delta(c))$

$= \tau(a)ba + \tau(a)bc + \tau(c)ba + \tau(c)bc + a\tau(b)a + a\tau(b)c + c\tau(b)a + c\tau(b)c + ab\,\delta(a) + cb\,\delta(a) + ab\,\delta(c) + cb\delta(c).$ (6)

From (5) & (6), we get

$\tau(a)ba + a\tau(b)a + ab\delta(a) + \delta(abc + cba) + \tau(c)bc + c\tau(b)c + cb\delta(c)$
$= \tau(a)ba + \tau(a)bc + \tau(c)ba + \tau(c)bc + a\tau(b)a + a\tau(b)c + c\tau(b)a + c\tau(b)c + ab\,\delta(a) + cb\,\delta(a) + ab\,\delta(c) + cb\delta(c)$

$\delta(abc + cba) = \tau(a)bc + a\tau(b)c + ab\,\delta(c) + \tau(c)ba + c\tau(b)a + cb\,\delta(a).$

***Lemma* 2**: [2] Let R be a semi prime ring if $axb = 0$ for all $x \in R$, then $bxa = 0$.

It will be convenient to denote $a^b = \delta(ab) - \tau(a)b - a\delta(b)$ and $[a,b] = ab - ba$.

We can easily, we have

$i) a^b = -b^a;$

$ii) a^{b+c} = a^b + a^c;$

$iii) (a+b)^c = a^c + b^c.$

***Theorem* 1**: If R is semiprime, then $a^b x\,[a,b] = 0$ for arbitrary $a, b, x \in R$.

Proof: Let $W = \delta(abxba + baxab),$

$W = \delta((ab)x(ba) + (ba)x(ab))$

On theone hand, we have

$W = \tau(ab)xba + ab\tau(x)ba + abx\delta(ba) + \tau(ba)xab + ba\tau(x)ab + bax\delta(ab)$

$W = (\tau(a)b + a\tau(b))xba + ab\tau(x)ba + abx\delta(ba) + (\tau(b)a + b\tau(a))xab + ba\tau(x)ab + bax\delta(ab)$

$W = \tau(a)bxba + a\tau(b)xba + ab\tau(x)ba + abx\delta(ba) + \tau(b)axba + b\tau(a)xab + ba\tau(x)ab + bax\delta(ab).$ (7)

On the other hand, we have

$W = \delta(a(bxb)a + b(axa)b)$
$= \tau(a)bxba + a\tau(bxb)a + abxb\delta(a) + \tau(b)axab + b\tau(axa)b + baxa\delta(b)$

$W = \tau(a)bxba + a\tau(b)xba + ab\tau(x)ba + abx\tau(b)a + abxb\delta(a) + \tau(b)axab + b\tau(a)xab + ba\tau(x)ab + bax\tau(a)b + baxa\delta(b).$ (8)

From equation (7) & (8), we get

$$\tau(a)bxba + a\tau(b)xba + ab\tau(x)ba + abx\delta(ba) + \tau(b)axba + b\tau(a)xab + ba\tau(x)ab + bax\delta(ab)$$
$$= \tau(a)bxba + a\tau(b)xba + ab\tau(x)ba + abx\tau(b)a + abxb\delta(a) + \tau(b)axab$$
$$+ b\tau(a)xab + ba\tau(x)ab + bax\tau(a)b + baxa\delta(b)$$
$abx\delta(ba) + bax\delta(ab) = abx\tau(b)a + abxb\delta(a) + bax\tau(a)b + baxa\delta(b)$
$abx((\delta(ba) - \tau(b)a - b\delta(a)) + bax(\delta(ab) - \tau(a)b - a\delta(b))$
$abxb^a + baxa^b = 0$
We know that $a^b = -b^a$
$(ba - ab)xa^b = 0$
$[a,b]x\, a^b = 0$
Since lemma 2, we have
$a^b x[a,b] = 0.$

***Corollary* 1:** If R is semiprime ring, then $a^b \in Z(R)$

Proof: For arbitrary $a,b,c,x \in R$,
By theorem 1, we have$a^{b+c}x[a,b+c] = 0$.
Again using theorem 1, we get
$a^b x[a,c] + a^c x[a,b] = 0$
$a^b x[a,c] = -a^c x[a,b]$ (9)
Multiplying (9) with $a^b x[a,c]y$ on both sides
And so $(a^b x[a,c]y(a^b x[a,c]) = -a^b x[a,c]ya^c x[a,b] = 0$
By lemma 2, we have $[a,c]ya^c = 0$
This implies that $a^b x[a,c] = 0$
Similarly, we have $a^b x[d,c] = 0$ for all $d \in R$
In particular $[a^b,c]x[a^b,c] = (a^b c - ca^b)x[a^b,c] = a^b(cx)[a^b,c] = 0$
This yields that $[a^b,c] = 0$
$\Longrightarrow a^b \in Z(R)$.

***Theorem* 2:**Let R be 2-torsion free prime ring, then every left generalized Jordan derivation on R is a leftgeneralized derivation.

Proof: Suppose that $\delta: R \to R$ is a left generalized Jordan derivation and τ is the relating Jordan derivation on R . By the proof of corollary 1, we have that $a^b x[c,d] = 0$ for all $a,b,c,d,x \in R$.We have two case

***Case*1** : *R is not commutative*

$$\textit{Then thereexists } c,d \in R \textit{ such that } [c,d] \neq 0 \textit{by the primness of } R, \textit{we conclude that } a^b$$
$$= 0, i.e\ \delta \textit{ is left generalized derivation}$$

***Case*2:**R is commutative
Let $w = \delta(a^2b + ba^2)$
$=\delta(a(ab) + (ba)a)$
$$= \tau(a)ab + a\delta(ab) + \tau(ba)a + ba\delta(a)$$
$W = \tau(a)ab + a\delta(ab) + \tau(b)a^2 + b\tau(a)a + ba\delta(a)$.(10)
On the other hand, we have
$W = \tau(a^2)b + a^2\delta(b) + \tau(b)a^2 + b\delta(a^2)$
$w = \tau(a)ab + a\tau(a)b + a^2\delta(b) + \tau(b)a^2 + b\tau(a)a + ba\delta(a)$(11)
From equation (10) &(11), we get
$$\tau(a)ab + a\delta(ab) + \tau(b)a^2 + b\tau(a)a + ba\delta(a)$$
$$= \tau(a)ab + a\tau(a)b + a^2\delta(b) + \tau(b)a^2 + b\tau(a)a + ba\delta(a)$$
$a\delta(ab) = a\tau(a)b + a^2\delta(b)$
$a\big(\delta(ab) - \tau(a)b - a\delta(b)\big) = 0$
$aa^b = 0$.
A linearization of the above expression with respect to a gives
$(a+c)(a+c)^b = o$
$(a+c)(a^b + c^b) = 0$
$aa^b + ac^b + ca^b + cc^b = 0$
We know that $aa^b = cc^b = 0$
$ac^b + ca^b = 0$.
$ac^b = -ca^b$
$ca^b yca^b = -ca^b yac^b = -ca^b ayc^b = -caa^b yc^b = 0$

$ca^b yca^b = 0$.
According to primness
$ca^b = 0$
$a^b ca^b = 0$
$a^b = 0$
$R$is left generalized derivation

***Lemma* 3**:For arbitrary$a, b, c \in R$ we have $\delta(abc + cba) = \tau(a)bc + a\tau(b)c + ab\delta(c) + \tau(c)ba + c\tau(b)a + cb\delta(a)$.

Proof:The similar argument as in the proof of lemma 1, we have
$W = \delta((a + c)b(a + c))$.
On the one hand,
$W = \tau(a + c)b(a + c) + (a + c)\tau(b)(a + c) + (a + c)b\delta(a + c)$
$= \big(\tau(a) + \tau(c)\big)(ba + bc) + \big(a\tau(b) + c\tau(b)\big)(a + c) + (ab + cb)(\delta(a) + \delta(c))$
$W = \tau(a)ba + \tau(a)bc + \tau(c)ba + \tau(c)bc + a\tau(b)a + a\tau(b)c + c\tau(b)a + c\tau(b)c + +ab\delta(a) + ab\delta(c) + cb\delta(a) + cb\delta(c)$.(12)
On the other hand,
$W = \delta(ab + cb)(a + c)$
$= \delta(aba + abc + cba + cbc)$
$= \delta(aba) + \delta(abc + cba) + \delta(cbc)$
$W = \tau(a)ba + a\tau(b)a + ab\delta(a) + \delta(abc + cba) + \tau(c)bc + c\tau(b)c + cb\delta(c)$(13)
From (12) & (13) are equal, we get
$\tau(a)ba + \tau(a)bc + \tau(c)ba + \tau(c)bc + a\tau(b)a + a\tau(b)c + c\tau(b)a + c\tau(b)c + +ab\delta(a) + ab\delta(c)$
$+cb\delta(a) + cb\delta(c) = \tau(a)ba + a\tau(b)a + ab\delta(a) + \delta(abc + cba) + \tau(c)bc + c\tau(b)c + cb\delta(c)$
$\delta(abc + cba) = \tau(a)bc + a\tau(b)c + ab\delta(c) + \tau(c)ba + c\tau(b)a + cb\delta(a)$.
For the purpose of this section we shall write $A(abc) = \delta(abc) - \tau(a)bc - a\tau(b)c - ab\delta(c)$ and $B(abc) = abc - cba$
We list a few elementary properties of $A$and B:
$i)A(abc) + A(cba) = 0$;
$ii)A\big((a + b)cd\big) = A(acd) + A(bcd)$and $B\big((a + b)cd\big) = B(acd) + B(bcd)$;
$iii)A(a(b + c)d) = A(abd) + A(acd)$and $B(a(b + c)d) = B(abd) + B(acd)$;
$iv)A\big(ab(c + d)\big) = A(abc) + A(abd)$and $B\big(ab(c + d)\big) = B(abc) + B(abd)$.

Lemma 4: Let R be a semi prime ring, then for arbitrary $a, b, c, x \in R$ we have $B(abc)xA(abc) = 0$.

Proof: Let $W = \delta(abcxcba + cbaxabc)$
On the one hand,
$W = \tau(abc)xcba + abc\tau(x)cba + abcx\delta(cba) + \tau(cba)xabc + cba\tau(x)abc + cbax\delta(abc)$
$W = (\tau(a)bc + a\tau(b)c + ab\tau(c))xcba + abc\,\tau(x)cba + abcx\delta(cba) + (\tau(c)ba + c\tau(b)a + cb\tau(a))xabc + cba\tau(x)abc + cbax\delta(abc)$
$W = \tau(a)bcxba + a\tau(b)cxcba + ab\tau(c)xcba + abc\tau(x)cba + abcx\delta(cba) + \tau(c)baxabc + c\tau(b)axabc + cb\tau(a)xabc + cba\tau(x)ab + cbax\delta(abc)$.(14)
On the other hand,
$W = \delta(a(bcxcb)a + c(baxab)c)$
$= \tau(a)bcxcba + a\tau(bcxcb)a + abcxcb\delta(a) + \tau(c)baxabc + c\tau(baxab)c + cbaxab\delta(c)$
$W = \tau(a)bcxcba + a\tau(b)cxcba + ab\tau(c)xcba + abc\tau(x)cba + abcx\tau(c)ba + abcxc\tau(b)a + abcxcb\delta(a) + \tau(c)baxabc + c\tau(b)axabc + cb\tau(a)xabc + cba\tau(x)abc + cbax\tau(a)bc + +cbaxa\tau(b)c + cbaxab\delta(a)$. (15)
From equation (14) & (15), we get
$\tau(a)bcxba + a\tau(b)cxcba + ab\tau(c)xcba + abc\tau(x)cba + abcx\delta(cba) + \tau(c)baxabc + c\tau(b)axabc + cb\tau(a)xabc + cba\tau(x)ab + cbax\delta(abc) =$
$\tau(a)bcxcba + a\tau(b)cxcba + ab\tau(c)xcba + abc\tau(x)cba + abcx\tau(c)ba + abcxc\tau(b)a + abcxcb\delta(a) + \tau(c)baxabc + c\tau(b)axabc + cb\tau(a)xabc + cba\tau(x)abc + cbax\tau(a)bc + +cbaxa\tau(b)c + cbaxab\delta(a)$
$abcx\big(\delta(cba) - \tau(c)ba - c\tau(b)a - cb\delta(a)\big) + cbax(\delta(abc) - \tau(a)bc - a\tau(b)c - ab\delta(c) = 0$.
$abcxA(cba) + cbaxA(abc) = 0$.
From(i),we have $A(abc) + A(cba) = 0$
The above equation becomes

$-abcxA(abc) + cbaxA(abc) = 0$
$(abc - cba)xA(abc) = 0$
$B(abc)xA(abc) = 0.$

Theorem 3: Let R be a semi prime ring, then $A(abc)xB(rst) = 0$ holds for all $a, b, c, x, r, s, t \in R$.

Proof: By Lemma 4, we have
$B(abc)xA(abc) = 0$
Linearizing theabove equation, we have
$B(a + rbc)xA(a + rbc) = 0$
$\big(B(abc) + B(rbc)\big)x\big(A(abc) + A(rbc)\big) = 0$
$B(abc)xA(abc) + B(abc)xA(rbc) + B(rbc)xA(abc) + B(rbc)xA(rbc) = 0$
$B(rbc)xA(abc) + B(abc)xA(rbc) = 0.$
$B(rbc)xA(abc) = -B(abc)xA(rbc)$
$B(rbc)xA(abc)yB(rbc)xA(abc) = -B(abc)xA(rbc)yB(rbc)xA(abc) = 0 \forall y \in R$
By the semiprimenessof R, we have that $B(rbc)xA(abc) = 0$
Similarly we can get that $B(rsc)xA(abc) = 0$ and
Further more $B(rst)xA(abc) = 0$.

Corollary 2: If R is a prime ring and $Z(R) \neq \{0\}$, then $A(abc) \in Z(R)$ forall $a, b, c \in R$.

Proof: For arbitrary $a, b, c, x, r, s \in R$ we have
$B(A(abc)rs)xB(A(abc)rs) = (A(abc)rs - srA(abc))xB(A(abc)rs$
$= A(abc)rsxB(A(abc)rs - srA(abc)xB(A(abc)rs = 0$
Primeness of R yealds that
$B(A(abc)rs = 0$
i.e. $A(abc)rs = srA(abc)$.
Choose non zero $r_0 \in Z(R)$, then $A(abc)r_0 s = sr_0 A(abc) = sA(abc)r_0$,
Which implies that $A(abc)r_0 \in Z(R)$.
Since R is prime,
We can easily conclude that $A(abc) \in Z(R)$

***Theorem* 4:** Let R be a 2-torsion free prime ring, then every left generalized Jordan triple derivation on R is a leftgeneralized derivation.

***Proof*:** Suppose that $\delta: R \to R$ is a left generalized Jordan triple derivation and τ is the relating Jordan triple derivation on R. We have two cases

***Case* 1:** There exists $r, s, t \in R$ sush that $B(rst) \neq 0$
Theorem 3 and primeness of R yield that $A(abc) = 0$

Case 2: $B(rst) = 0$, holds for all $r, s, t \in R$
i.e. $rst = tsr$
Let Q be the central closer or the martandale right quotient ring of R (see [5] for the deffination) of R, then Q is a primering with identity and contains R. By [3] Q satisfies the same generalized polynomial identities as R. In particular $rst = tsr$ for all r, s, t in Q. Taking $s = 1$ yields the commutativity of Q and R.
Now let $w = \delta(a^3bc + cba^3)$
On the one hand,we have
$w = \tau(a^3)bc + a^3\tau(b)c + a^3b\delta(c) + \tau(c)ba^3 + c\tau(b)a^3 + cb\delta(a^3)$,
$w = \tau(a)a^2bc + a\tau(a)abc + a^2\tau(a)bc + a^3\tau(b)c + a^3b\delta(c) + \tau(c)ba^3 + c\tau(b)a^3 + cb\tau(a)a^2 + cba\tau(a)a + cba^2\delta(a)$. (16)
On the other hand, we have
$w = \delta(abcaa + aaabc)$
$w = \tau(abc)a^2 + abc\tau(a)a + abca\delta(a) + \tau(a)a^2bc + a\tau(a)abc + a^2\delta(abc)$
$w = \tau(a)bca^2 + a\tau(b)ca^2 + ab\tau(c)a^2 + abc\tau(a)a + abca\delta(a) + \tau(a)a^2bc + a\tau(a)abc + a^2\delta(abc)$. (17)
The two expressions yield that
$a^2[\delta(abc) - \tau(a)bc - a\tau(b)c - ab\delta(a)] = 0$
i.e $a^2A(abc) = 0$
Then we have $(A(abc)a)x(A(abc)a)$
$A(abc)a^2x(A(abc) = o$
Hence $A(abc)a = 0$,

Furthermore .with same approach in the proof of the theorem 2, we obtain $A(abc)x = 0$and so $A(abc)xA(abc) = 0$ which implies that $A(abc) = 0$
It remains to prove that δ is a left generalized derivation.
Again, let a, b, c, d, x, y be arbitrary elements of R and
$w = \delta(abxab)$.
On the one hand, we have
$w = \tau(ab)xab + ab\tau(x)ab + abx\delta(ab),$
On the other hand, we have
$w = \tau(a)bxab + a\tau(bxa)b + abxa\delta(b).$
Then we have
$abx\big(\delta(ab) - \tau(a)b - a\delta(b)\big) = 0$
By the notation $a^b = \delta(ab) - \tau(a)b - a\delta(b),$ we get
$abxa^b = 0.$
Furthermore $(a + c)bx(a + c)^b = 0$
This shows that $cbxa^b = -abxc^b$ and
$(cbxa^b)y(cbxa^b) = -abx(c^b ycb)xa^b,$
Hence $cbxa^b = 0$.
Similarly we have $cdxa^b = 0$
In particular $ca^b xca^b = 0$
And so $ca^b = 0$, which leads to$a^b = 0$.
Every left generalized Jordan triple derivation on 2-torsion free semi prime ring is a left generalized derivation.

REFERENCES

1. M.Bresar,Jordan mappings of semiprime rings, J.Algebra 17(1989),218-228.
2. M.Bresar,Jordan derivations on semiprime rings, Proc.Amer. Math.Soc.104 (1988).
3. I.N.Herstein, Jordan derivation of prime rings,Prco.Amar.Math.Soc. 8(1957),1104-1110.
4. B.Havla, Generalized derivation in rings,Comm.Algebra 26(1998),1147-1166.
5. J.Zhu and C.Xiong, Generalized derivations on rings and mappings of P-preserving kernel into range on von Nemann algebras, Acta Math. Sinica (in Chinese) 41(1998), 795-800.
6. Wu jing and shijie Lu, Generalized Jordan derivations on prime rings and standard operatoralgebra,vol. No.4,pp., december 2003, 605-613.